In conclusion, we should note that, as in [9], in this work the effect of kinetic processes (the nonequilibrium nature of the vibrational and rotational degrees of freedom, condensation) and the effect of rarefaction, in the broad sense of these concepts, were not examined. Moreover, the starting parameters were chosen in such a way that the kinetic processes indicated had no significant effect on the nature of the interaction.

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BURGERS APPROXIMATION FOR PLANE LONG-WAVELENGTH DISTURBANCES IN AEROSUSPENSIONS

S. V. Tarakanov and O. M. Todes

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In reports devoted to the investigation of transient normal compression shocks in aerodisperse media (see [1, 2], for example), essentially only the initial stage of evolution of these shocks, the section of formation of the relaxation wave, is considered. The study of subsequent stages in the evolution of shock waves in aerosuspensions is of no less interest from the point of view of assuring industrial safety in connection with performing explosive work (mining, explosive welding, etc.). In the present report the final stage of evolution of waves in suspensions of solid and liquid particles, the stage of degeneration of a shock wave into a sound wave, is investigated by methods of nonlinear acoustics. The dissipative properties of aerosuspensions are analyzed and the structure of the shock front in this stage of evolution is examined.

We will assume that solid or liquid particles with a constant weight concentration are suspended in an inert gas in the initial stage undisturbed by the wave. Before the arrival of the wave the aerosuspension is assumed to be monodisperse, at rest in the coordinate system (x, t), and in equilibrium. In the case of liquid particles the gaseous phase contains,

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in addition to the inert gas, vapors of the liquid with a partial pressure equal to the saturation pressure at the temperature of the drops. The flow of the aerodisperse medium will be described using a model of interpenetrating continuous media.

First we show that the equations of dynamics of the two-continuum model developed in [3-6] and elsewhere are reduced to a Burgers equation for the velocity of the gaseous phase for weak long-wavelength disturbances. We note that the equations obtained in [7, 8] are not valid in the case of disturbances of long duration, since they do not change into the equations of the equilibrium model.

1. The Long-Wavelength Approximation and the Burgers Approximation for an Aerosuspension of Solid Particles

Without imposing limits on the intensity of the wave, we consider only those simplifications which follow from the assumption that the characteristic times of relaxation of the particles are small compared with the duration of the wave process.

The system of equations describing an aerosuspension of solid particles can be represented in the form (see [9], for example)

$$d_t\rho + \rho\partial_x u = 0,\ p = \rho T; \tag{1.1}$$

$$\rho d_t u + \frac{a_0^2}{\gamma}\partial_x p = r\frac{v-u}{\tau_u}; \tag{1.2}$$

$$\rho d_t T + (\gamma - 1)p\partial_x u = r\chi(\Theta - T)/\tau_T + \gamma(\gamma-1)(v-u)^2/\tau_u; \tag{1.3}$$

$$d_{2t}r + r\partial_x v = 0; \tag{1.4}$$

$$d_{2t}v = (u - v)/\tau_u,\ d_{2t}\Theta = (T - \Theta)/\tau_T, \tag{1.5}$$

where $d_t = \partial_t + u\partial_x$; $d_{2t} = \partial_t + v\partial_x$; ρ, r, u, v, T, and Θ are the densities, velocities, and temperatures of the gaseous and disperse phases; p is the pressure; γ is the adiabatic index of the gaseous phase; a_0 is the frozen-in speed of sound; τ_u and τ_T are the dynamic and thermal relaxation times; χ is the ratio of specific heats of the particles and gas at constant volume.

Equations (1.1)-(1.5) are written in dimensionless form. As the scales of pressure, density, and temperature we take the values of the respective parameters in the undisturbed gas. As the units of measurement of velocity, time, and length we take the equilibrium speed of sound $a_e = [g/(1+\varkappa)]^{0.5}a_0$ ($\varkappa$ is the initial weight concentration of the disperse phase and $g = (\gamma + \varkappa\chi)(1 + \varkappa\chi)^{-1}$), the characteristic time τ_0 of the wave process, and the length $a_e\tau_0$, respectively.

Equations (1.5) can be integrated along the trajectories of motion $y(t)$ of particles of the disperse phase, where $d_t y = v$. Assuming that $v = 0$ and $\Theta = 1$ at $t = t_0$, we will have

$$v = \tau_u^{-1}\int_{t_0}^{t}\exp\left(\frac{\vartheta - t}{\tau_u}\right)u\,[y(\vartheta), \vartheta]\,d\vartheta,$$

$$\Theta = \tau_T^{-1}\int_{t_0}^{t}\exp\left(\frac{\vartheta - t}{\tau_T}\right)T\,[y(\vartheta), \vartheta]\,d\vartheta + \exp\left(\frac{t_0 - t}{\tau_T}\right).$$

In the long-wavelength approximation τ_u, $\tau_T \ll 1$, so that the integrands near the upper limit make the main contribution to the integrals. Expanding $u(\vartheta)$ and $T(\vartheta)$ in series near $\vartheta = t$ on this basis and taking τ_u and τ_T as quantities of the same order, we obtain

$$v = u - \tau_u d_t u + \tau_u^2\left(d_t u\partial_x u + d_{tt}^2 u\right) + O\left(\tau_u^3\right), \tag{1.6}$$

$$\Theta = T - \tau_T d_t T + \tau_T^2\left(d_t u\partial_x T + d_{tt}^2 T\right) + O\left(\tau_T^3\right).$$

Substituting Eqs. (1.6) into (1.2)-(1.4) and discarding terms containing the square of the relaxation times, we have

$$\begin{gathered}(\rho + r)\, d_t u + a_0^2 \gamma^{-1} \partial_x p = r\tau_u \left(d_t u \partial_x u + d_{tt}^2 u\right), \\ (\rho + r\chi)\, dT_t + (\gamma - 1)\, p\partial_x u = r\chi\tau_T \left(d_t u \partial_x T + d_{tt}^2 T\right) + \gamma(\gamma - 1)\, r\tau_u (d_t u)^2, \\ d_t r + r\partial_x u = \tau_u \partial_x (r d_t u).\end{gathered} \tag{1.7}$$

Thus, in the long-wavelength approximation the initial system (1.1)-(1.5) was reduced to the system (1.1), (1.7). Since terms containing the square of the characteristic relaxation times were omitted in Eqs. (1.7), for rough estimates of the applicability of this approximation we can use the condition τ_u^2, $\tau_T^2 \ll 1$. Further simplification can be obtained for wave processes of still greater duration (τ_u, $\tau_T \ll 1$).

In fact, taking the relaxation time to zero in (1.7), with allowance for (1.1) we obtain a closed system describing the equilibrium flow of the aerosuspension (see [9], for example). It is obvious that for waves of infinite duration (an infinite step) the equilibrium description is exact, since $\tau_u = \tau_T = 0$ in this case. Such a description allows one to determine the propagation velocity of the shock wave and the equilibrium parameters of the medium behind the relaxation zone, although it provides no information about the structure of the shock front, its width, etc. In a first approximation this information can be obtained on the basis of the model (1.1), (1.7). For further refinement of the picture of the relaxation processes one must allow for higher-order terms in the expansions (1.6) or else one must integrate the initial system (1.1)-(1.5).

Now let us proceed to the consideration of weak waves. We introduce disturbances of the parameters of the aerosuspension in the form $p' = p - 1$, $\rho' = \rho - 1$, $T' = T - 1$, $u' = u$, $r' = (r - \varkappa)/\varkappa$ and we will assume that these disturbances are quantities of first-order smallness $\sim O(\varepsilon)$. We also assume that the relaxation times τ_u and τ_T are $\lesssim O(\varepsilon)$, while the concentration $\varkappa$ of the disperse phase is $\lesssim O(1)$. Under these assumptions we reduce the system (1.1), (1.7) to one equation using Khokhlov's method [10], according to which the parameters of the wave in the comoving coordinate system ($\xi = t - x$, t) will be slowly varying functions of time t because of the weak nonlinearity. On this basis, following [10], we henceforth assume that in the variables (ξ, t) time differentiation increases the order of smallness by one.

Changing to the coordinates (ξ, t) in (1.1) and (1.7), eliminating p', ρ', r', and T' successively from the analysis, and discarding terms of $\sim O(\varepsilon^3)$, we obtain the Burgers equation for the velocity of the gaseous phase,

$$\partial_t u - k u \partial_\xi u = \eta \partial_{\xi\xi}^2 u, \tag{1.8}$$

where $k = 0.5(g + 1)$, $\eta = \eta_u + \eta_T$, and

$$\eta_u = \frac{\tau_u}{2} \frac{\varkappa}{1 + \varkappa}, \quad \eta_T = \frac{\tau_T}{2} \frac{\varkappa\chi (g - 1)}{g (1 + \varkappa\chi)}. \tag{1.9}$$

Thus, for a weak disturbance ($u \ll 1$) of long duration (τ_u^2, $\tau_T^2 \ll 1$) an aerosuspension of solid particles represents a viscous medium. The development of viscosity, as follows from (1.9), is due to processes of friction (η_u) and heat exchange (η_T) between phases.

2. The Burgers Approximation for an Aerosuspension of Liquid Particles

To describe wave processes in drop suspensions we use the system of equations [9]

$$\begin{gathered}d_t \rho_1 + \rho_1 \partial_x u = 0, \; d_t \rho_2 + \rho_2 \partial_x u = j, \\ \rho d_t u + a_0^2 \gamma^{-1} \partial_x p = f + (v - u)\, j, \\ (\rho_1 \chi_1 + \rho_2 \chi_2) d_t T + (\gamma - 1) p \partial_x u = q + \gamma(\gamma - 1)(v - u) f \\ + [\gamma_2 \chi_2 \Theta - \chi_2 T + 0.5\gamma(\gamma - 1)(v - u)^2] j, \\ p = p_1 + p_2, \; p_i = m_i \rho_i T, \; m_i = \mu/\mu_i, \; \rho = \rho_1 + \rho_2, \; d_{2t} r + r\partial_x v = -j, \; d_{2t} S = -\frac{2}{3}\frac{S}{r} j, \\ r d_{2t} v = -f, \; r\chi d_{2t} \Theta = -q - \gamma H j, \\ j = \frac{r}{S\tau_D} \frac{p_{2e} - p_2}{p}, \; f = \frac{r}{S\tau_u} (v - u), \; q = \frac{r\chi}{S\tau_T} (\Theta - T), \\ p_{2e} = p_{20} \exp\left[\frac{\gamma H}{(\gamma - 1)\, m_2} (1 - \Theta^{-1})\right],\end{gathered} \tag{2.1}$$

where ξ is the molecular weight; S is the surface area of a particle; $H = h_e/c_{p0}T_0$ is the dimensionless heat of vaporization; τ_D is the diffusional relaxation time; the indices 1 and 2 mark parameters of the inert gas and the vapor; the variables ρ, u, p, T, μ, and γ without indices correspond to the gaseous phase.

The quantities appearing in Eqs. (2.1) are made dimensionless exactly as in Sec. 1. We only note that the equilibrium speed of sound a_e is not known in advance, so that the quantity $a_0^2\gamma^{-1}$ remains undetermined as yet.

We introduce small departures $O(\varepsilon)$ of the parameters of the medium from the equilibrium parameters through the equations $\rho_i' = \rho_i - \varkappa_i$, $p_i' = p_i - m_i\varkappa_i$ $(i = 1,\ 2)$, $T' = T - 1$, $r' = (r - \varkappa)/\varkappa$, $S' = 1 - S$, $\Theta' = \Theta - 1$, $u' = u$, and $v' = v$. The letters $\varkappa_1$ and $\varkappa_2$ denote the concentrations of the inert gas and the vapor in the equilibrium state, so that $\varkappa_1 + \varkappa_2 = \varkappa_1 m_1 + \varkappa_2 m_2 = \varkappa_1\chi_1 + \varkappa_2\chi_2 = 1$. We assume that the concentration $\varkappa$ of the disperse phase is $\lesssim O(1)$. For simplicity we will henceforth use a linear dependence of the saturated vapor pressure p_{2e}' on the temperature Θ', p_{2e}', where $z = \varkappa_2\gamma H(\gamma - 1)^{-1}$.

Substituting the expressions for the perturbations into (2.1) and changing to the comoving coordinates $(\xi,\ t)$, with the accuracy of $O(\varepsilon^2)$, inclusively, we obtain (here and later the primes to the variables are omitted)

$$\begin{gathered}
\partial_t\rho_1 + (1 - u)\partial_\xi\rho_1 - (\varkappa_1 + \rho_1)\partial_\xi u = 0,\ \partial_t\rho_2 \\
+ (1 - u)\,\partial_\xi\rho_2 - (\varkappa_2 + \rho_2)\,\partial_\xi u = \varkappa(1 + r + S - p)(z\Theta - p_2)/\tau_D, \\
\partial_t u + (1 + \rho_1 + \rho_2 - u)\,\partial_\xi u - a_0^2\gamma^{-1}\partial_\xi p = \varkappa\,(1 + r + S)\,\frac{v - u}{\tau_u}, \\
\partial_t T + (1 + \rho_1\chi_1 + \rho_2\chi_2 - u)\partial_\xi T - (\gamma - 1)(1 + p)\partial_\xi u = \varkappa\chi\,(1 + r + S) \\
\times\,(\Theta - T)/\tau_T + \varkappa\chi_2(\gamma_2 - 1)(1 + r + S + T - p)(z\Theta - p_2)/\tau_D,\ p = (m_1\rho_1 \\
+ m_2\rho_2)(1 + T) + T,\ p_2 = m_2\rho_2(1 + T) + m_2\varkappa_2 T,\ \partial_t r + (1 - v)\partial_\xi r - (1 + r) \\
\times\,\partial_\xi v = -(1 + r + S - p)(z\Theta - p_2)/\tau_D,\quad \partial_t S + (1 - v)\partial_\xi S = (2/3)\ (1 \\
- p)\,(z\Theta - p_2)/\tau_D,\ \partial_t v + (1 - v)\partial_\xi v = (1 + S)(u - v)/\tau_u, \\
\partial_t\Theta + (1 - v)\,\partial_\xi\Theta = (1 + S)\left[\frac{T - \Theta}{\tau_T} - \frac{\gamma H}{\chi}(1 - p)\,\frac{z\Theta - p_2}{\tau_D}\right].
\end{gathered} \tag{2.2}$$

Integrating the last two equations of the system (2.2), we find the connection between the velocity and temperature of the particles and the remaining parameters of the aerosuspension with the required accuracy,

$$\begin{gathered}
v = u - \tau_u\partial_t u - \tau_u(1 - u - S)\,\partial_\xi u + \tau_u^2\partial_{\xi\xi}^2 u + O(\varepsilon^4), \\
\Theta = T - \tau_*\partial_t\varphi - \tau_*(1 - u - S + \alpha p)\,\partial_\xi\varphi + \tau_*^2\partial_{\xi\xi}^2\varphi + \frac{\alpha}{z}p\psi + O(\varepsilon^4).
\end{gathered} \tag{2.3}$$

where

$$\begin{gathered}
\varphi = (1 - \alpha)(T + Hp_2);\quad \psi = (1 - \alpha)(zT - p_2),\quad \tau_* = (1 - \alpha)\,\tau_T. \\
\alpha = zH(1 + zH)^{-1}.
\end{gathered}$$

Eliminating the vapor pressure and the temperature T from (2.2), as well as the particle velocity and temperature in accordance with (2.3), we obtain a system of six differential and two algebraic equations describing the evolution of the perturbations in an aerosuspension under consideration. Let us reduce the system thus obtained to one equation.

Since we intend to bring out primarily the qualitative relationships of the influence of mass exchange on the parameters of waves propagating in an aerosuspension below, we assume that $\varkappa_2 \ll 1$. This corresponds physically to the case in which the temperature of the medium is considerably lower than the boiling temperature of the liquid. In addition, to simplify the expressions obtained below we will discard a number of unimportant terms. The relative contribution of these terms was estimated from the parameters of water drops in air at atmospheric pressure and a temperature of 20°C (in this case $\varkappa_2 \approx 0.02$). Thus, in the expression for g (see (2.4)) we neglect the work of pressure forces, $\chi_2(\gamma_2 - 1)$, connected with the change in the volume of the substance in the phase transition, in comparison with the heat γH of the phase transition, etc. We note that the accuracy of the equations thus obtained increases with a decrease in the initial temperature of the medium, so that in the case of water drops in air at atmospheric pressure the description given below is valid with variation of the temperature from 0 to ≈20°C, i.e., it covers the most important temperature range for practical work. Evidently, it should be expected that the qualitative character of the results is retained in a somewhat wider temperature range.

We determine the connection of the parameters of an aerosuspension with the velocity of the gaseous phase in the acoustical approximation and find the value of the coefficient $a_0^2\gamma^{-1}$. With allowance for the adopted assumptions, we have

$$\frac{\gamma(1+\varkappa)}{a_0^2} = g = \frac{1}{c}\left(\gamma + \varkappa\chi + \frac{z}{m_2}\gamma H\right).$$
$$c = 1 + \varkappa\chi + \frac{z}{m_2}\gamma H,\ \varphi = \frac{\gamma - 1}{c}u = (g-1)\,u.$$
$$\rho_1 = \varkappa_1 u,\ \rho_2 = \frac{z}{m_2}(g-1)\,u.$$
$$r = \left[1 - \frac{\varkappa_2}{\varkappa}\left(\frac{\gamma H}{m_2 c} - 1\right)\right]u,\ S = \frac{2}{3}\frac{\varkappa_2}{\varkappa}\left(\frac{\gamma H}{m_2 c} - 1\right)u.$$
$$\frac{\psi}{\tau_D} = \left[\frac{\varkappa_2}{\varkappa}\left(\frac{\gamma H}{m_2 c} - 1\right) + z(1-\alpha)\frac{\chi}{\gamma}(g-1)\right]\partial_\xi u. \tag{2.4}$$

Using Eqs. (2.4) we represent the equation of motion of the gaseous phase of the system (2.2) in the form

$$\partial_t u + \partial_\xi u - \frac{1}{g}\partial_\xi p = \frac{\varkappa}{1+\varkappa}\tau_u\partial^2_{\xi\xi}u. \tag{2.5}$$

From this it follows that to reduce the initial system (2.2) to one equation it remains to find an expression for $\partial_\xi p$ through derivatives of the velocity u with the accuracy of terms $O(\varepsilon^2)$, inclusively.

Eliminating $\partial_\xi p$ from (2.5) using the remaining equations of the system (2.2), we obtain Eq. (1.8), the coefficients k and η of which are defined as

$$k = 0.5(g+1),\ \eta = \eta_u + \eta_T + \eta_D, \tag{2.6}$$
$$\eta_u = \frac{\tau_u}{2}\frac{\varkappa}{1+\varkappa},\ \eta_T = \frac{\tau_T}{2}\frac{g-1}{g}\frac{\varkappa\chi}{c\left[1 + \varkappa_2\gamma H^2(\gamma-1)^{-1}\right]},$$
$$\eta_D = \frac{\tau_D}{2}\left\{\varkappa_2\frac{\chi H\left(\gamma H^2 m_2^{-1}c^{-1} - 1\right)}{gc\left[1 + \varkappa_2\gamma H^2(\gamma-1)^{-1}\right]} + \frac{\varkappa_2}{\varkappa}\left[\left(\frac{\gamma H}{m_2 c}\right)^2 H - \frac{\gamma H}{m_2 c}(H+1) + 1\right]\right\}.$$

As follows from (2.6), the effective viscosity of an aerosuspension of liquid particles is due to friction (η_u, heat exchange (η_T), and mass exchange (η_D) between phases. We note that in the limit $\varkappa_2 \to 0$ the expressions (2.6) change into Eqs. (1.9) for an aerosuspension of solid particles.

In the derivation of Eq. (1.8) from the system (2.1) the disturbances of all the parameters of the medium were assumed to be small. However, the change in particle size will be small only at high enough values of the concentration of the disperse phase, i.e., when $\varkappa > \varkappa_m$. The value of $\varkappa_m$ can be estimated from the relation $2\varkappa_2\left(\gamma H m_2^{-1}c^{-1} - 1\right) \approx 3\varkappa$ (see (2.4)), from which we obtain $\varkappa_m \approx 0.1$ for an aerosuspension of water drops in air under the conditions indicated above.

3. Dissipative Properties of Aerosuspensions

As shown above, for weak long-wavelength disturbances the interaction between the gaseous and disperse phases can be allowed for using coefficients of viscosity, the expressions for which through the parameters of the aerosuspension are determined by Eqs. (1.9) and (2.6). From these equations it follows that the effective viscosity is proportional to the characteristic times of the relaxation processes. Therefore, in the equilibrium approximation (τ_u, τ_T, $\tau_D \to 0$) the evolution of disturbances is described by the equation

$$\partial_t u - ku\partial_\xi u = 0. \tag{3.1}$$

It is known (see [11], for example) that the solution of Eq. (1.8) as $\eta \to 0$ comes down to the discontinuous solution of Eq. (3.1) satisfying the condition $U = -ku/2 = -(g+1)u/4$

at the discontinuity, where U is the velocity of movement of the discontinuity. Consequently, the model under consideration is a generalization of an equilibrium approximation. From the physical point of view the limiting transition τ_u, τ_T, $\tau_D \to 0$ corresponds either to a decrease in the diameter of particles of the disperse phase (τ_u, τ_T, $\tau_D \sim D^2$) or to an increase in the duration of the wave process.

Let us consider the influence of the concentration $\varkappa$ of the dispersed substance on the coefficients of viscosity of the aerosuspension. In Fig. 1 we present the dependence of $b = 2\eta\tau_u^{-1}$ on $\varkappa$ for suspensions of solid (solid lines $\chi = 1$, dot–dash lines $\chi = 4.2$) and liquid (dashed lines $\chi = 4.2$, $H = 6.6$, $m_2 = 1.6$) particles in air ($\gamma = 1.4$). It is assumed that $\tau_u = \gamma\chi^{-1}\tau_T = \tau_D$. Lines 0-3 correspond to the total ($b = b_u + b_T + b_D$), "dynamic," "thermal," and "diffusional" viscosities. It is seen that in an aerosuspension of solid particles with $\chi = 1$ the "dynamic" viscosity considerably exceeds the "thermal" viscosity in practically the entire range of concentrations, i.e., in the given case friction between the phases plays the decisive role in processes of interaction of the phases. With an increase in the heat capacity χ the role of heat exchange increases considerably, as seen from Fig. 1, especially in the region of low concentrations. We note that since the adiabatic index approaches unity with an increase in $\varkappa$ the "thermal" viscosity varies nonmonotonically. Its maximum value of $b_T = \chi(1 - \gamma^{-1})(1 + \gamma^{0.5})^{-2}$ is reached at $\varkappa = \gamma^{0.5}\chi^{-1}$. Despite this, the total viscosity grows monotonically with an increase in concentration.

The dissipative properties of an aerosuspension change considerably when mass exchange between the phases is present. In this case, along with the "dynamic" and "thermal" viscosities, the ratio between which remains approximately the same as for solid particles (see Fig. 1), "diffusional" viscosity connected with phase transitions is also present in the medium. With a decrease in concentration the quantity b_D increases, and formally $b_D \to \infty$ as $\varkappa \to 0$. As was noted, however, the region of small $\varkappa$ ($\varkappa \leqslant 0.1$) for an aerosuspension of liquid particles cannot be analyzed within the framework of the present model. Such behavior of the coefficient b_D leads to nonmonotonic variation of the total viscosity of an aerosuspension of liquid particles (see Fig. 1).

The following physical explanation can be given for the increase in "diffusional" viscosity with a decrease in the concentration of the disperse phase. The process of equalization of the temperatures of the gaseous and disperse phases can be divided into two stages. In the first stage the particle temperature rises in a time on the order of τ_T to the value at which all the heat supplied to the particle is expended on the phase transition. After this, since the particle temperature hardly varies now, equalization of the temperatures of the two phases is accomplished through variation of the gas temperature. From the equation of energy of the gaseous phase (see (2.2)) it is seen that the characteristic time of this process is $\tau_T(\varkappa\chi)^{-1}$.

The good agreement between the total viscosities of aerosuspensions of liquid and solid particles at the same values of the heat capacity of the disperse phase ($\chi = 4.2$) and $\varkappa > 0.6$ (see Fig. 1) should be noted. This means that at relatively high concentrations of the dispersed substance the role of mass exchange is insignificant compared with the force and thermal interactions of the phases.

4. Structure and Evolution of Waves in an Aerosuspension

Solutions of the model problems under consideration here for the Burgers equation are well known (see [11], for example), so that we turn attention only to their specifics in application to an aerosuspension.

To investigate the structure of a wave we consider the problem of the decay of an infinite step. Let a disturbance $u(\xi, 0) = V\vartheta(\xi)$ be assigned at the initial time. Then the solution (1.8) at $t \gg 1$ is written in the form

$$u = V[1 + \exp(-y\mathrm{Re})]^{-1},$$

where $\mathrm{Re} = 0.5kV\eta^{-1}$ is the Reynolds number; $y = \xi + 0.5kVt$ is the spatial variable connected with the wavefront.

We find the thickness λ of the wavefront by defining it as $\lambda = V(\partial_y u)^{-1}|_{y=0}$. We will have

$$\lambda = 4\mathrm{Re}^{-1} = 16\eta V^{-1}(g + 1)^{-1}.$$

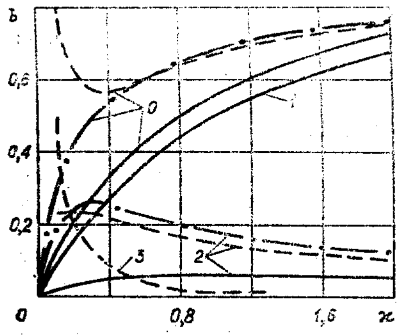

Fig. 1

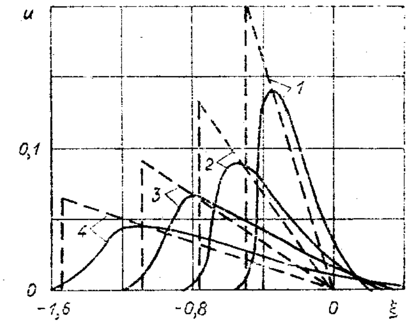

Fig. 2

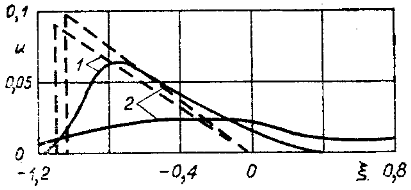

Fig. 3

Thus, because of the linear dependence of λ on viscosity we can speak of different thicknesses λ_u, λ_T, and λ_D of the wavefront connected with processes of friction, heat exchange, and mass exchange between phases, and since the quantity g depends weakly on the parameters of the aerosuspension, the character of the variation of these thicknesses is fully determined by the behavior of the coefficients of viscosity corresponding to them.

As was noted, more precise information about the structure of the wave can be obtained if the next discarded terms are allowed for in the expansions (1.6) and (2.3). This would lead to the appearance not only of dissipation but also of dispersion in the medium. A whole series of investigations have been devoted to the analysis of the dispersion properties of media with relaxation (see [12], for example). The results obtained show that allowance for dispersion terms usually does not lead to significant corrections.

Turning to an analysis of the evolution of a wave of finite duration, we note that the initial shape of the disturbances plays a pronounced role only in the early stage of their evolution, i.e., as estimates show, in the time interval of $(0, \max(\xi_0^2\eta^{-1}, \xi^2\eta^{-1}\mathrm{Re}^{-1}))$, where ξ_0 is the size of the disturbed region at t = 0. Therefore, to establish the asymptotic characteristics (at $t \gg 1$) of the damping the initial condition is unimportant. Assigning this condition in the form of a δ impulse $(u(\xi, 0) = A\delta(\xi))$, we will have

$$u = \frac{2A}{(\pi \,\mathrm{Re}\, kt)^{0.5}} \frac{(e^{\mathrm{Re}} - 1)\, e^{-0.5\zeta^2}}{e^{\mathrm{Re}} + 1 + 2\,(e^{\mathrm{Re}} - 1)\,\Phi(\zeta)}, \tag{4.1}$$

where $\Phi(\zeta)$ is the probability integral; $\mathrm{Re} = kA(2\eta)^{-1}$; $\zeta = \xi(2\eta t)^{-0.5}$.

The solution of this problem in the equilibrium approximation ($\eta = 0$) has the form

$$\xi_f = -(2Akt)^{0.5}, \; u_f = (2A/kt)_{0.5}, \; u = -\xi/kt, \; \xi \in (\xi_f, 0) \tag{4.2}$$

and represents the classical Crussard solution.

An analysis shows (see [11]) that at small values of the viscosity the solution (4.1) corresponds basically to the equilibrium solution (4.2) except for sections with lengths of $\sim Re^{-1}$ and $Re^{-0.5}$ located near strong and weak discontinuities. For an aerosuspension of solid particles this means that at low concentrations of the dispersed substance the relaxation processes resulting in the "slippage" of the gaseous and disperse phases with respect to velocity and temperature play a role only in these sections. With an increase in concentration the "slippage" effects become important over the entire extent of the wave disturbance. In Fig. 2 we present the evolution of a wave (A = 0.05) in an aerosuspension of solid particles ($\varkappa=0.05$, $\chi=1$, $\tau_u=0.1$) with and without allowance for relaxation processes (solid lines for solution (4.1) and dashed lines for (4.2)). Lines 1-4 correspond to times t = 2, 5, 10, and 20. As seen from Fig. 2, "slippage" of the phases leads to smearing out of the wave and a decrease in its amplitude compared with an equilibrium wave when relaxation processes are ignored. The role of relaxation processes grows still more with a further increase in the concentration of the disperse phase. In Fig. 3 we give profiles of waves with $\varkappa = 0.05$ and 0.5 (lines 1 and 2, respectively) at the time t = 10; the values of the remaining parameters are the same as in Fig. 2. It is seen that with $\varkappa = 0.5$ the wave does not resemble an equilibrium wave even remotely. This occurs because at high enough values of η the viscous term in Eq. (1.8) becomes dominant over the nonlinear term. Then the solution of the problem of the evolution of a δ impulse is described by the expression

$$u = A(4\pi\eta t)^{-0.5} \exp(-0.5\zeta^2), \tag{4.3}$$

which represents the source function for the one-dimensional equation of heat conduction. Just such a case occurs in practice in Fig. 3 (solid curve 2).

Equally high values of the coefficient of "effective" viscosity are characteristic of an aerosuspension of liquid particles, as follows from Fig. 1. Therefore, the evolution of a wave disturbance in an aerosuspension of liquid particles with $\tau_u = 0.1$ is also described by Eq. (4.3).

The above analysis showed that relaxation processes of exchange of mass, momentum, and heat between phases lead to smearing out of the wave profile (see Figs. 2 and 3). And within the framework of the given approximation the influence of each of the indicated processes is additive ($\eta = \eta_u + \eta_T + \eta_D$ and $\lambda = \lambda_u + \lambda_T + \lambda_D$). The effect of smearing out of the wave is the greater, the higher the values of the coefficients of "viscosity," the connection of which with the parameters of the medium is given by Eqs. (1.9) and (2.6). We note that nonuniformity of the aerosuspension must be taken into account even at very low values of η; for the cases presented in Figs. 2 and 3, $\eta \approx 3\cdot10^{-3}$ and $2\cdot10^{-2}$.

The role of processes of interaction between phases is not uniform. For a suspension of solid particles with $\chi = 1$, for example, momentum exchange plays the dominant role in the entire accessible range of concentrations, while allowance for heat exchange leads to approximately a 10% correction to the determination of η (see Fig. 1). In the case of liquid particles (water drops) at low concentrations ($\varkappa_m < \varkappa \lesssim 0.5$) the effects of friction and heat and mass exchange are comparable with each other (see Fig. 1), while at higher concentrations mass exchange becomes unimportant compared with the force and thermal interactions of the phases.

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VORTICAL SINGULARITY OF SELF-SIMILAR GAS FLOW IN THE REFLECTION OF A SHOCK WAVE

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We consider the problem of the transient gas flow developing when a plane shock wave, propagating through a stationary homogeneous gas 0 and characterized by a Mach number $M = u/a_0$, strikes a rigid impermeable wall forming a wedge with an angle θ at the time $t = 0$. We consider the case in which the angle θ is large enough that the wave configuration represented in Fig. 1 developed in the interaction, where CD is the incident wave, BC is the regularly reflected wave, BE is the front of the rarefaction wave propagating from AO through the region of uniform flow 2, AB is the diffracted wave, and 3 is the region of nonuniform flow. In the absence of a characteristic size the problem is self-similar with arguments $x = X/t$ and $y = Y/t$.

In the case of $\theta \to \pi/2$ [1, 2], $u = v = 0$ at point O, i.e., the flow is stagnant. An asymptotic analysis as $\theta \to 0$ indicates that in the vicinity of point O flow takes place around the corner with conservation of most of the X component of the velocity. One can assume that in the intermediate case of $\theta < \pi/2$ flow around the corner takes place near point O and gas particles which have entered 3 through point A on the diffracted shock wave are located at the wall OE (at some part of it) adjacent to point O. On the other hand, obviously, particles which have entered 3 from 2 under the action of the rarefaction wave BE are located on the part of the wall OE adjacent to point E. The point separating the gas which has reached OE in different ways is designated as F in Fig. 1.

If one assumes that the gas is nonviscous and thermally nonconducting, then one can state that the entropy per particle is conserved during motion inside 3, and therefore particles on different sides of point F have different entropies and, because of the continuity of pressure, different densities. Point F, a vortical flow singularity [3, 4] of the type under consideration, corresponds to a Furry singularity of conical gas flows [5].

The vortical singularity (VS) at point F is an essential element of the fundamental problem of reflection of a shock wave from a wedge under consideration. In each of the approaches (analytical, numerical, or experimental) the motifs connected with its existence must be taken into account. Very satisfactory results obtained using a variant of a numeri-

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